

Supersymmetric non-Abelian noncommutative Chern-Simons theory

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Abstract

In this work, we study the three-dimensional non-Abelian noncommutative supersymmetric Chern-Simons model with the $U(N)$ gauge group. Using a superfield formulation, we prove that, for the pure gauge theory, the Green functions are one-loop finite in any gauge, if the gauge superpotential belongs to the fundamental representation of $u(N)$; this result also holds when matter in the fundamental representation is included. However, the cancellation of both ultraviolet and ultraviolet/infrared divergences only happens in a special gauge if the coupling of the matter is in the adjoint representation. We also look into the finite one-loop quantum corrections to the effective action: in the pure gauge sector the Maxwell together with its corresponding gauge fixing action are generated; in the matter sector, the Chern-Simons term is generated, inducing a shift in the classical Chern-Simons coefficient.

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I. INTRODUCTION

The noncommutative Chern-Simons theory has been intensively studied during the last years. Among the most important results obtained so far are the vanishing of the ultraviolet/infrared (UV/IR) infrared divergences for the pure Chern-Simons theory [1] and the quantization of the Chern-Simons coefficient [2, 3, 4, 5]. There were also some indications that, in the axial gauge, the pure Chern-Simons theory is actually a free field theory [6]. Besides all these theoretical developments, a relation of noncommutative Chern-Simons theory to the fractional quantum Hall effect has also been pointed [7].

In our previous paper [8], we investigated various interactions of the noncommutative Chern-Simons field with the matter with special concern on the existence of dangerous UV/IR singularities. For the supersymmetric model, we found that the cancellation of UV/IR infrared divergences occurs even when matter is included, either in the adjoint or in the fundamental representation (in the former case, this happens only in a particular gauge).

In this paper, we continue our line of research by looking at two natural extensions of our previous results. First, we extend the investigations of [8] to the non-Abelian case. As we shall prove, similarly to other four- and three-dimensional noncommutative supersymmetric gauge theories [9, 10], the cancellation of UV/IR singularities, generated in the nonplanar part of the quantum corrections, demands that the gauge group generators are in the fundamental representation of the $U(N)$ group. We recall that this is the same requirement found for the consistency of noncommutative gauge models at the classical level [11]. Second, we will calculate the finite one-loop quantum corrections to the effective action of the supersymmetric theory (see [12] for a similar calculation in the non-supersymmetric case, as well as [13] for an extensive study of the commutative theory). In particular, this will allow us to determine the finite changes in the coefficient of the Chern-Simons term and the structure of the effective action of the model.

Our work is organized as follows. In the next section we demonstrate the finiteness of the pure gauge sector if the generators are in the fundamental representation of $u(N)$. The effect of the inclusion of matter is considered in section III where we prove that finiteness in the adjoint representation also requires the gauge superfield to be in a special gauge. The one loop calculation of the finite contributions to the quadratic part of the effective action is done in section IV. There, we prove that nonlocal Maxwell and Chern-Simons terms are

generated, and the later one induces a shift in the classical Chern-Simons coefficient. In the summary, section V, we present our concluding remarks.

II. FINITENESS IN THE PURE GAUGE SECTOR

The action of the three-dimensional noncommutative non-Abelian Chern-Simons theory with an arbitrary gauge group is $S_{total} = S_{CS} + S_{GF} + S_{FP}$ [14]. Here, S_{CS} is the classical action

$$S_{CS} = \frac{m}{g^2} \int d^5 z \text{Tr} \left(A^\alpha * W_\alpha + \frac{i}{6} \{A^\alpha, A^\beta\} * D_\beta A_\alpha + \frac{1}{12} \{A^\alpha, A^\beta\} * \{A_\alpha, A_\beta\} \right), \quad (2.1)$$

S_{GF} and S_{FP} are the gauge fixing and Faddeev-Popov actions

$$S_{GF} = -\frac{m}{2g^2\xi} \int d^5 z \text{Tr} (D^\alpha A_\alpha) * (D^\beta A_\beta) \quad (2.2)$$

and

$$S_{FP} = \frac{1}{2g^2} \int d^5 z \text{Tr} (c' * D^\alpha D_\alpha c + i c' * D^\alpha [A_\alpha, c]), \quad (2.3)$$

where

$$W_\beta = \frac{1}{2} D^\alpha D_\beta A_\alpha - \frac{i}{2} [A^\alpha, D_\alpha A_\beta] - \frac{1}{6} [A^\alpha, \{A_\alpha, A_\beta\}] \quad (2.4)$$

is the superfield strength constructed from the Lie-algebra valued spinor superpotential $A_\alpha \equiv A_\alpha^a T^a$, and the T^a are the generators of the corresponding gauge group in a given representation. They satisfy the normalization condition $\text{Tr}(T^a T^b) = \kappa \delta^{ab}$. Hereafter it is implicitly assumed that all commutators and anticommutators involve both algebraic and Moyal (anti)commutation. As usual, in this work we consider only space-space noncommutativity, to evade unitarity problems [15].

The action (2.1) is invariant under the BRST transformations

$$\delta A_\alpha = -\epsilon \nabla_\alpha c = -\epsilon (D_\alpha c + i[A_\alpha, c]), \quad (2.5)$$

$$\delta c = \epsilon c^2, \quad \delta c' = \frac{\epsilon}{\xi} D_\alpha A^\alpha, \quad (2.6)$$

where ϵ is an infinitesimal Grassmannian gauge parameter. The quadratic part of the action fixes the gauge propagator to be

$$\langle A^{a\alpha}(z_1) A^{b\beta}(z_2) \rangle = \frac{i g^2}{4m\kappa \square} \delta^{ab} [D^\beta D^\alpha + \xi D^\alpha D^\beta] \delta^5(z_1 - z_2), \quad (2.7)$$

where $C^{\alpha\beta} = -C_{\alpha\beta}$ is the second-rank antisymmetric Levi-Civita symbol defined with the normalization $C^{12} = i$. The ghost fields are also Lie-algebra valued and their propagator is given by

$$\langle c'^a(z_1) c^b(z_2) \rangle = -i g^2 \delta^{ab} \frac{D^2}{\kappa \square} \delta^5(z_1 - z_2). \quad (2.8)$$

The interaction part of the classical action in the pure gauge sector is

$$\begin{aligned} S_{int} = \frac{m}{g^2} \int d^5 z \operatorname{Tr} \bigg(& -\frac{i}{2} A^\alpha * [A^\beta, D_\beta A_\alpha] - \frac{1}{6} A^\alpha * [A^\beta, \{A_\beta, A_\alpha\}] \\ & + \frac{i}{6} \{A^\alpha, A^\beta\} * D_\beta A_\alpha + \frac{1}{12} \{A^\alpha, A^\beta\} * \{A_\alpha, A_\beta\} \bigg), \end{aligned} \quad (2.9)$$

from which we get the Feynman supergraphs vertices

$$\begin{aligned} V_3 &= -\frac{i m}{3 g^2} A^{a\alpha}(k_1) A^{b\beta}(k_2) D_\beta A_\alpha^c(k_3) (A^{abc} e^{ik_2 \wedge k_3} - A^{acb} e^{ik_3 \wedge k_2}), \\ V_4 &= -\frac{m}{6 g^2} A^{a\alpha}(k_1) A^{b\beta}(k_2) A_\alpha^c(k_3) A_\beta^d(k_4) [e^{i(k_1 \wedge k_2 + k_3 \wedge k_4)} A^{abcd} - e^{i(k_1 \wedge k_3 + k_2 \wedge k_4)} A^{acbd}], \end{aligned} \quad (2.10)$$

where we have introduced the notation

$$A^{ab\dots c} \equiv \operatorname{Tr}(T^a T^b \dots T^c). \quad (2.11)$$

The ghost-gauge field vertex has the following form,

$$V_c = \frac{i}{2g^2} D^\alpha c'^a(k_1) A_\alpha^b(k_2) c^c(k_3) (A^{abc} e^{ik_2 \wedge k_3} - A^{acb} e^{ik_3 \wedge k_2}). \quad (2.12)$$

We are now in a position to study the divergences in the one-loop quantum corrections to the effective action. The non-Abelian structure of the gauge group does not change

the power counting we found for the Abelian theory [8], so that the superficial degree of divergence for a supergraph with E external field legs and N_D covariant derivatives acting on those legs is $\omega = 2 - \frac{1}{2}E - \frac{1}{2}N_D$. It follows that the linear UV divergences are possible only for supergraphs with two external gauge legs. As in [10, 16], logarithmic divergences are absent due to symmetric integration on the loop momentum. Therefore, the complete one-loop analysis of the divergences of the (pure gauge) model demands the evaluation of the diagrams depicted in Fig. 1.

For the sake of simplicity, we only give the details of the evaluation of the diagram 1b. One starts by taking into account all contractions of two fields in the non-symmetric vertex V_4 in Eq. (2.10), that is to say,

$$\begin{aligned} \Gamma_{1b}(p) = & -\frac{m}{6g^2} \int \left(\prod \frac{d^3 k_i}{(2\pi)^3} \right) d^2\theta \left\{ e^{i(k_1 \wedge k_2 + k_3 \wedge k_4)} A^{abcd} - e^{i(k_1 \wedge k_3 + k_2 \wedge k_4)} A^{acbd} \right\} \times \\ & \left[+ A_\alpha^c(k_3) A_\beta^d(k_4) \langle A^{\alpha a}(k_1) A^{\beta b}(k_2) \rangle + A_\alpha^c(k_3) A_\beta^b(k_2) \langle A^{\alpha a}(k_1) A^{\beta d}(k_4) \rangle \right. \\ & + A_\alpha^a(k_1) A_\beta^d(k_4) \langle A^{\alpha c}(k_3) A^{\beta b}(k_2) \rangle + A_\alpha^a(k_1) A_\beta^b(k_2) \langle A^{\alpha c}(k_3) A^{\beta d}(k_4) \rangle \\ & \left. - A^{\alpha b}(k_2) A_\alpha^d(k_4) \langle A^{\beta a}(k_1) A_\beta^c(k_3) \rangle - A^{\alpha a}(k_1) A_\alpha^c(k_3) \langle A^{\beta b}(k_2) A_\beta^d(k_4) \rangle \right]. \end{aligned} \quad (2.13)$$

The six terms in Eq. (2.13) are divided in two groups of similar contributions. After some manipulations one finds, for the first four terms in the square bracket of Eq. (2.13),

$$\begin{aligned} \Gamma_{1b}^{(1)}(p) = & -\frac{1}{24\kappa} \int \frac{d^3 k}{(2\pi)^3} d^2\theta \left[(A^{abcc} - A^{bacc}) + 2A^{abcc} - 2\cos(2k \wedge p) A^{acbc} \right] \times \\ & (\xi D^\alpha D^\beta + D^\beta D^\alpha) \delta_{11} A_\alpha^a(p, \theta) A_\beta^b(-p, \theta), \end{aligned} \quad (2.14)$$

while, for the remaining two terms,

$$\begin{aligned} \Gamma_{1b}^{(2)}(p) = & \frac{1}{24\kappa} \int \frac{d^3 k}{(2\pi)^3} d^2\theta \left[-2A^{abcc} + 2\cos(2k \wedge p) A^{acbc} \right] \times \\ & (\xi D^\alpha D_\alpha + D_\alpha D^\alpha) \delta_{11} A^{\beta a}(p, \theta) A_\beta^b(-p, \theta). \end{aligned} \quad (2.15)$$

One readily realizes that the factor $(A^{abcc} - A^{bacc})$ inside the square brackets of Eq. (2.14) do not contribute for symmetry reasons. After D -algebra manipulations (which are trivial

in this case), one can cast the contribution to the two-point vertex function of the gauge superpotential corresponding to the diagram 1b as

$$\Gamma_{1b}(p) = \frac{1}{4\kappa} (1 - \xi) \int \frac{d^3k}{(2\pi)^3} d^2\theta \left[A^{abcc} - 2 \cos(2k \wedge p) A^{abc} \right] A^{\alpha a}(p, \theta) A_{\alpha}^b(-p, \theta) . \quad (2.16)$$

As for the other diagrams in Fig. 1, their complete evaluation follows along the same lines but is far more intricate. Here, we just quote their divergent parts,

$$\Gamma_{1a}(p) = \frac{1}{4\kappa^2} \xi \int \frac{d^3k}{(2\pi)^3} d^2\theta \left[A^{cad} A^{dbc} - A^{cad} A^{cbd} \cos(2k \wedge p) \right] \frac{1}{k^2} A^{\alpha a}(p, \theta) A_{\alpha}^b(-p, \theta), \quad (2.17a)$$

$$\Gamma_{1c}(p) = -\frac{1}{4\kappa^2} \int \frac{d^3k}{(2\pi)^3} d^2\theta \left[A^{cad} A^{dbc} - A^{cad} A^{cbd} \cos(2k \wedge p) \right] \frac{1}{k^2} A^{\alpha a}(p, \theta) A_{\alpha}^b(-p, \theta), \quad (2.17b)$$

postponing the detailed discussion of their finite parts to the Section IV.

The planar part of $\Gamma_1 = \Gamma_{1a} + \Gamma_{1b} + \Gamma_{1c}$, being proportional to $\int \frac{d^3k}{k^2}$, exactly vanishes within the framework of dimensional regularization, which we are implicitly assuming. As for the nonplanar part of Γ_1 , which would generate a linear UV/IR infrared divergence, its cancellation is secured by the condition involving traces of the gauge group generators,

$$\frac{1}{\kappa} A^{cad} A^{cbd} - A^{acbc} = 0, \quad (2.18)$$

which is satisfied for the $U(N)$ gauge group in the fundamental representation. Equation (2.18) is the same condition we found for the cancellation of dangerous UV/IR infrared singularities for the three- and four-dimensional noncommutative super-Yang-Mills theories [9, 10]. Again, we see that the strong restriction imposed on the choice of the gauge group at the classical level [11] also plays an outstanding role to enforce the absence of singularities in the quantum theory.

As a final remark, by means of the substitution $A^{ab\dots c} \rightarrow 1$ in Eqs. (2.16) and (2.17), we correctly reobtain the results we have found for the Abelian theory in [8].

III. INTERACTION WITH MATTER

We will study now the consequences of the addition of matter in the theory. Matter can interact with the Chern-Simons superfield in two different ways. It can be in the fundamental representation,

$$S_{\text{matter}}^{Fund} = \int d^5z \text{Tr} \left(\bar{\phi}(D^2 - M)\phi - \frac{i}{2}(\bar{\phi} * A^\alpha * D_\alpha \phi - D^\alpha \bar{\phi} * A_\alpha * \phi) - \frac{1}{2}\bar{\phi} * A^\alpha * A_\alpha * \phi \right), \quad (3.1)$$

or in the adjoint representation,

$$S_{\text{matter}}^{Adj} = \int d^5z \text{Tr} \left(\bar{\phi}(D^2 - M)\phi - \frac{i}{2}([\bar{\phi}, A^\alpha] * D_\alpha \phi - D^\alpha \bar{\phi} * [A_\alpha, \phi]) - \frac{1}{2}[\bar{\phi}, A^\alpha] * [A_\alpha, \phi] \right). \quad (3.2)$$

In the presence of matter, the power counting for the supergraphs remains the same as before, but E must be considered as the total number of external legs. Therefore, only supergraphs with two external legs can generate nonintegrable (linear) UV/IR infrared divergences.

The supergraphs with a matter internal loop and external gauge legs (see Fig. 2) are identical to the ones studied in [10], where their total contribution was proved to be free of nonintegrable UV/IR infrared divergences if and only if the condition (2.18) was satisfied. We also note that all UV divergent terms in these diagrams cancel due to the relation

$$\frac{1}{\kappa} A^{cad} A^{dac} = A^{abcc}, \quad (3.3)$$

which also happens to hold in the fundamental representation of $u(N)$. Therefore we need to study only the graphs with external matter legs.

First, we consider matter in the fundamental representation, as specified in Eq. (3.1). One can show that all the relevant diagrams are totally planar and therefore generate no UV/IR mixing. The cancellation of the UV divergences in these planar parts follows the same pattern as in the previous section.

When matter is in the adjoint representation, we find the following divergent contributions from the supergraphs depicted in Figs. 3a and 3b,

$$\begin{aligned}
\Gamma_{3a}(p) &= -\frac{i\xi g^2}{2m\kappa^2} \int \frac{d^3k}{(2\pi)^3} d^2\theta \frac{1}{k^2} [A^{cad}A^{dbc} - A^{cad}A^{cbd} \cos(2k \wedge p)] \phi^a(-p, \theta) \bar{\phi}^b(p, \theta), \\
\Gamma_{3b}(p) &= \frac{i(1-\xi)g^2}{2m\kappa} \int \frac{d^3k}{(2\pi)^3} d^2\theta \frac{1}{k^2} [A^{abcc} - A^{acbc} \cos(2k \wedge p)] \phi^a(-p, \theta) \bar{\phi}^b(p, \theta). \quad (3.4)
\end{aligned}$$

Again, the planar parts of these contributions vanish within the framework of dimensional regularization. The nonplanar parts would generate dangerous UV/IR mixing, but they cancel if the relation

$$\frac{1}{\kappa} A^{cad} A^{cbd} \xi - A^{acbc} (1 - \xi) = 0 \quad (3.5)$$

is obeyed. The condition (3.5) is satisfied only in the gauge $\xi = 1/2$ where it reproduces Eq. (2.18). Therefore, as observed before, the cancellation only happens in the fundamental representation of $u(N)$. We remind the reader that the Abelian theory is also free of nonintegrable UV/IR singularities in the gauge $\xi = 1/2$ [8]. For a different value of ξ , the matter two-point function develops an UV/IR infrared divergence, even if the pure gauge sector is infrared safe, as we have shown in the previous section.

IV. FINITE CONTRIBUTION TO THE TWO-POINT VERTEX FUNCTIONS

After having demonstrated the one-loop finiteness of the model, in this section we determine the finite one-loop quantum corrections to the quadratic part of the effective action. A clear importance of this result is the possibility to verify whether in the quantum theory a shift in the Chern-Simons coefficient is produced.

In the pure gauge sector, the two-point vertex function of the gauge superpotential receives finite contributions only from the diagrams *a* and *c* in Fig. 1. The more complicated diagram is *1a*, since there are 18 different ways to contract two pairs of superfields to form the internal lines. We classify these contributions according to the distribution of supercovariant derivatives, namely $\Gamma_a = \Gamma_{aI} + \Gamma_{aII} + \Gamma_{aIII} + \Gamma_{aIV}$ where, after simplifications of the Moyal phase factors,

$$\begin{aligned}
\Gamma_{aI}(p) = & \frac{m^2}{9} \int \frac{d^3k}{(2\pi)^3} d^2\theta_1 d^2\theta_2 F_L^{ab}(k, p) \times \\
& \left[+ A^{\alpha a}(-p, \theta_1) A^{\beta' b}(p, \theta_2) \langle A^\beta(1) A^{\alpha'}(2) \rangle \langle D_\alpha A_\beta(1) D_{\alpha'} A_{\beta'}(2) \rangle \right. \\
& + A^{\alpha a}(-p, \theta_1) A^{\alpha' b}(p, \theta_2) \langle A^\beta(1) A^{\beta'}(2) \rangle \langle D_\alpha A_\beta(1) D_{\alpha'} A_{\beta'}(2) \rangle \\
& + A^{\beta a}(-p, \theta_1) A^{\beta' b}(p, \theta_2) \langle A^\alpha(1) A^{\alpha'}(2) \rangle \langle D_\alpha A_\beta(1) D_{\alpha'} A_{\beta'}(2) \rangle \\
& \left. + A^{\beta a}(-p, \theta_1) A^{\alpha' b}(p, \theta_2) \langle A^\alpha(1) A^{\beta'}(2) \rangle \langle D_\alpha A_\beta(1) D_{\alpha'} A_{\beta'}(2) \rangle \right], \quad (4.1)
\end{aligned}$$

$$\begin{aligned}
\Gamma_{aII}(p) = & -\frac{m^2}{9} \int \frac{d^3k}{(2\pi)^3} d^2\theta_1 d^2\theta_2 F_L^{ab}(k, p) \times \\
& \left[+ A^{\alpha a}(-p, \theta_1) A^{\beta' b}(p, \theta_2) \langle A^\beta(1) D_{\alpha'} A_{\beta'}(2) \rangle \langle D_\alpha A_\beta(1) A^{\alpha'}(2) \rangle \right. \\
& + A^{\alpha a}(-p, \theta_1) A^{\alpha' b}(p, \theta_2) \langle A^\beta(1) D_{\alpha'} A_{\beta'}(2) \rangle \langle D_\alpha A_\beta(1) A^{\beta'}(2) \rangle \\
& + A^{\beta a}(-p, \theta_1) A^{\beta' b}(p, \theta_2) \langle A^\alpha(1) D_{\alpha'} A_{\beta'}(2) \rangle \langle D_\alpha A_\beta(1) A^{\alpha'}(2) \rangle \\
& \left. + A^{\beta a}(-p, \theta_1) A^{\alpha' b}(p, \theta_2) \langle A^\alpha(1) D_{\alpha'} A_{\beta'}(2) \rangle \langle D_\alpha A_\beta(1) A^{\beta'}(2) \rangle \right], \quad (4.2)
\end{aligned}$$

$$\begin{aligned}
\Gamma_{aIII}(p) = & \frac{m^2}{9} \int \frac{d^3k}{(2\pi)^3} d^2\theta_1 d^2\theta_2 F_L^{ab}(k, p) \times \\
& \left[+ D_\alpha A_\beta^a(-p, \theta_1) D_{\alpha'} A_{\beta'}^b(p, \theta_2) \langle A^\alpha(1) A^{\alpha'}(2) \rangle \langle A^\beta(1) A^{\beta'}(2) \rangle \right. \\
& \left. + D_\alpha A_\beta^a(-p, \theta_1) D_{\alpha'} A_{\beta'}^b(p, \theta_2) \langle A^\alpha(1) A^{\beta'}(2) \rangle \langle A^\beta(1) A^{\alpha'}(2) \rangle \right], \quad (4.3)
\end{aligned}$$

and

$$\begin{aligned}
\Gamma_{aIV}(p) = & \frac{2m^2}{9} \int \frac{d^3k}{(2\pi)^3} d^2\theta_1 d^2\theta_2 F_L^{ab}(k, p) \times \\
& \left[+ D_\alpha A_\beta^a(-p, \theta_1) A^{\beta' b}(p, \theta_2) \langle A^\alpha(1) A^{\alpha'}(2) \rangle \langle A^\beta(1) D_{\alpha'} A_{\beta'}(2) \rangle \right. \\
& + D_\alpha A_\beta^a(-p, \theta_1) A^{\alpha' b}(p, \theta_2) \langle A^\alpha(1) A^{\beta'}(2) \rangle \langle A^\beta(1) D_{\alpha'} A_{\beta'}(2) \rangle \\
& + D_\alpha A_\beta^a(-p, \theta_1) A^{\beta' b}(p, \theta_2) \langle A^\beta(1) A^{\alpha'}(2) \rangle \langle A^\alpha(1) D_{\alpha'} A_{\beta'}(2) \rangle \\
& \left. + D_\alpha A_\beta^a(-p, \theta_1) A^{\alpha' b}(p, \theta_2) \langle A^\beta(1) A^{\beta'}(2) \rangle \langle A^\alpha(1) D_{\alpha'} A_{\beta'}(2) \rangle \right]. \quad (4.4)
\end{aligned}$$

The Moyal phase factor common to all these diagrams is given by

$$F_L^{ab}(k, p) = (A^{cad}A^{dbc} - A^{cad}A^{cbd} \cos 2k \wedge p) , \quad (4.5)$$

and, in a slight abuse of notation, factors like $\langle A_\alpha(1)A_\beta(2) \rangle$ in Eqs. (4.1)-(4.4) mean the gauge superpropagator in Eq. (2.7) without the δ^{ab} , which has already been used to simplify the phase factor $F_L^{ab}(k, p)$.

Finally, the graph in Fig. 1c yields the ghost contribution,

$$\Gamma_c(p) = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} d^2\theta_1 d^2\theta_2 F_L^{ab}(k, p) A_\beta^a(-p, \theta_1) A_\beta^b(p, \theta_2) \times \\ [D_1^\alpha D^2 \delta^2(\theta_1 - \theta_2)] [D^2 D_2^\beta \delta^2(\theta_1 - \theta_2)] . \quad (4.6)$$

Only the contributions Γ_{aI} and Γ_c have divergent parts, which were already discussed in Section III. For the remaining of this section, we restrict ourselves to the discussion of the finite parts of Γ_a and Γ_c . To evaluate them, one has to perform rather lengthy D -algebra manipulations, thus generating lots of terms. These were calculated independently by hand and also by means of a computer program designed for quantum superfield calculations [17].

We do not quote all the details of the calculation, but we will describe the necessary steps using Γ_{aI} as a typical example of the contributions we have evaluated. First of all, using D -algebra manipulations, we can cast Γ_{aI} as follows,

$$\Gamma_{aI}^{Fin}(p) = -\frac{1}{144} \int \frac{d^3k}{(2\pi)^3} d^2\theta \frac{F_L^{ab}(k, p)}{k^2(p-k)^2} (k_{\alpha\delta} C_{\beta\gamma} + k_{\beta\gamma} C_{\alpha\delta}) \times \\ \left[(D^\beta D^\gamma + \xi D^\gamma D^\beta) A^{\alpha a}(p, \theta) A^{\delta b}(-p, \theta) + (D^\alpha D^\delta + \xi D^\delta D^\alpha) A^{\beta a}(p, \theta) A^{\gamma b}(-p, \theta) + \right. \\ \left. + (D^\beta D^\delta + \xi D^\delta D^\beta) A^{\alpha a}(p, \theta) A^{\gamma b}(-p, \theta) + (D^\alpha D^\gamma + \xi D^\gamma D^\alpha) A^{\beta a}(p, \theta) A^{\delta b}(-p, \theta) \right] , \quad (4.7)$$

where the superscript *Fin* is to stress that we are quoting only the finite part. Next, we introduce the definition

$$I^{ab}(p) = \int \frac{d^3k}{(2\pi)^3} \frac{F_L^{ab}(k, p)}{k^2(k+p)^2} , \quad (4.8)$$

and verify that

$$\int \frac{d^3 k}{(2\pi)^3} \frac{F_L^{ab}(k, p)}{k^2(p-k)^2} k_{\alpha\beta} = \frac{p_{\alpha\beta}}{2} I^{ab}(p), \quad (4.9)$$

to write

$$\begin{aligned} \Gamma_{aI}^{Fin} = & \frac{1}{36} \int \frac{d^3 p}{(2\pi)^3} d^2 \theta I^{ab}(p) \left[p^2 A^{\alpha a}(p, \theta) A_\alpha^b(-p, \theta)(1 + \xi) \right. \\ & \left. + p_{\alpha\beta} D^2 A^{\alpha a}(p, \theta) A^{\beta b}(-p, \theta)(1 - \xi) \right]. \end{aligned} \quad (4.10)$$

The final form of Γ_{aI}^{Fin} and is obtained by means of the identities [14]

$$\begin{aligned} \int \frac{d^3 p}{(2\pi)^3} d^2 \theta p^2 A^{\gamma a}(p, \theta) A_\gamma^b(-p, \theta) &= \int \frac{d^3 p}{(2\pi)^3} d^2 \theta \left[-2 (L_{Maxw})^{ab} + 2 (L_{MGF})^{ab} \right], \\ \int \frac{d^3 p}{(2\pi)^3} d^2 \theta p_{\beta\gamma} D^2 A^{\beta a}(p, \theta) A^{\gamma b}(-p, \theta) &= \int \frac{d^3 p}{(2\pi)^3} d^2 \theta \left[2 (L_{Maxw})^{ab} + 2 (L_{MGF})^{ab} \right], \end{aligned} \quad (4.11)$$

where

$$(L_{Maxw})^{ab} = \frac{1}{2} W_0^{\alpha a} W_{0\alpha}^b, \quad (L_{MGF})^{ab} = \frac{1}{4} D^\beta A_\beta^a(p, \theta) D^2 D^\gamma A_\gamma^b(-p, \theta) \quad (4.12)$$

are the Maxwell (“linearized Yang-Mills”) and the gauge-fixing Lagrangians of the three-dimensional SYM theory, respectively. Also, $W_0^{\beta a} \equiv \frac{1}{2} D^\alpha D^\beta A_\alpha^a$ is the linear part of the superfield strength defined in (2.4).

Using the steps just described, we obtained the finite parts of Γ_a and Γ_c as follows,

$$\begin{aligned} \Gamma_{aI}^{Fin} &= \frac{1}{9} \int \frac{d^3 p}{(2\pi)^3} d^2 \theta I^{ab}(p) \left\{ \xi (L_{Maxw})^{ab} - (L_{MGF})^{ab} \right\}, \\ \Gamma_{aII}^{Fin} &= \frac{1}{36} \int \frac{d^3 p}{(2\pi)^3} d^2 \theta I^{ab}(p) \left\{ \xi(\xi + 4) \left[(L_{Maxw})^{ab} - (L_{MGF})^{ab} \right] \right\}, \\ \Gamma_{aIII}^{Fin} &= -\frac{1}{36} \int \frac{d^3 p}{(2\pi)^3} d^2 \theta I^{ab}(p) \left\{ (1 - \xi) \left[4\xi (L_{Maxw})^{ab} + (1 - \xi) (L_{MGF})^{ab} \right] \right\}, \\ \Gamma_{aIV}^{Fin} &= -\frac{1}{9} \int \frac{d^3 p}{(2\pi)^3} d^2 \theta I^{ab}(p) \left\{ (1 - \xi) \left[\xi (L_{Maxw})^{ab} + (1 + \frac{1}{2}\xi) (L_{MGF})^{ab} \right] \right\}, \\ \Gamma_c^{Fin} &= \frac{1}{4} \int \frac{d^3 p}{(2\pi)^3} d^2 \theta I^{ab}(p) \left\{ (L_{Maxw})^{ab} - (L_{MGF})^{ab} \right\}. \end{aligned} \quad (4.13)$$

As a result, the total pure gauge contribution to the two-point gauge superpotential function is

$$\Gamma_{2\text{ gauge}} = \frac{1}{4} \int \frac{d^3 p}{(2\pi)^3} d^2 \theta I^{ab}(p) \left[(1 + \xi^2) (L_{Maxw})^{ab} - 2 (L_{MGF})^{ab} \right]. \quad (4.14)$$

Therefore, we find that both nonlocal Maxwell and the gauge-fixing actions are generated by quantum corrections, and there is no gauge in which any of them vanish. Note that the Chern-Simons term is not generated here.

The only source for the Chern-Simons term as a quantum correction is the coupling to the matter (see Fig. 2) in which case we obtained the following contribution to the two-point gauge superpotential function [18],

$$\begin{aligned} \Gamma_{2\text{ matter}} = & -\frac{1}{2} \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 k}{(2\pi)^3} d^2 \theta J^{ab}(k, p) (k_{\gamma\beta} + M C_{\gamma\beta}) \\ & \left[(D^2 A^{\gamma a}(-p, \theta)) A^{\beta b}(p, \theta) + \frac{1}{2} D^\gamma D^\alpha A_\alpha^a(-p, \theta) A^{\beta b}(p, \theta) \right], \end{aligned} \quad (4.15)$$

where

$$J^{ab}(k, p) = \frac{1}{(k^2 + M^2)[(k - p)^2 + M^2]} \begin{bmatrix} A^{cad} A^{dbc} \\ 2F_L^{ab}(k, p) \end{bmatrix}. \quad (4.16)$$

In Eq. (4.16), the upper (lower) row corresponds to the fundamental (adjoint) representation for the matter field. An equivalent form for $\Gamma_{2\text{ matter}}$ is

$$\Gamma_{2\text{ matter}} = \frac{1}{4} \int \frac{d^3 k}{(2\pi)^3} d^2 \theta f^{ab}(p) [W_0^{\alpha a} W_{0\alpha}^b + 2M W_0^{\alpha a} A_\alpha^b], \quad (4.17)$$

where

$$f^{ab}(p) = \int \frac{d^3 k}{(2\pi)^3} J^{ab}(k, p). \quad (4.18)$$

We stress that the simplicity of Eq. (4.17) depends on the relation (3.3), which enforces the cancellation of additional finite terms that would appear in Eq. (4.17) if the generators T^a were not in the fundamental representation of $u(N)$.

The above results show explicitly the generation of nonlocal Maxwell and Chern-Simons terms. From Eq. (4.17), we find that the radiative corrections to the original Chern-Simons coefficient m/g^2 turn out to be

$$\frac{1}{2}Mf^{ab}(0) = \frac{N}{16\pi}\varepsilon(M) \left[\begin{array}{c} A^{cad}A^{dbc} \\ 2F_L^{ab}(p=0) \end{array} \right]. \quad (4.19)$$

Here, ε is the sign function. Recalling the completeness relation satisfied by the generators of the $U(N)$ group in the fundamental representation, $(T^a)_{ij}(T^a)_{kl} = \kappa\delta_{il}\delta_{jk}$, and that the value of κ in this representation is $1/2$, one obtains

$$\frac{1}{2}Mf^{ab}(0) = \frac{N}{128\pi}\varepsilon(M) \left[\begin{array}{c} \delta^{ab} \\ 2(\delta^{ab} - \delta^{a0}\delta^{b0}) \end{array} \right]. \quad (4.20)$$

For the matter in the fundamental representation, the above result is similar to the ones found in the literature for nonsupersymmetric theories [3, 4], where a shift proportional to $N\varepsilon(M)$ was also found. On the other hand, when matter is in the adjoint representation, the shift only appears for the non-Abelian components of the gauge superpotential. This is consistent with the absence of this shift in the Abelian theory, as can be seen from our calculations in [8].

V. SUMMARY

We studied the quantum dynamics of the noncommutative supersymmetric Chern-Simons theory with an arbitrary gauge group, in the one-loop approximation. Our first result consists in the fact that the cancellation of the nonintegrable (linear) UV/IR infrared divergences occurs only if the generators of the gauge group are in the fundamental representation of $U(N)$. This condition is the same as the one found for the four-dimensional super-Yang-Mills [9] and the three-dimensional super-Yang-Mills [10] theories in spite of the radical difference between their superfield formulations.

We have also calculated the finite one-loop corrections to the gauge superpotential two-point function, and we verified that a nonlocal Maxwell and the corresponding gauge fixing terms are generated from the pure gauge sector. We note that none of them vanish in any gauge. From the matter radiative corrections, besides the nonlocal Maxwell, a Chern-Simons term is also induced. If the matter is in the fundamental representation, there is a

shift in the classical Chern-Simons coefficient proportional to $N \varepsilon(M)$. Interestingly enough, for the matter in the adjoint representation, the Chern-Simons coefficient receives quantum corrections only for the non-Abelian components of the gauge superpotential.

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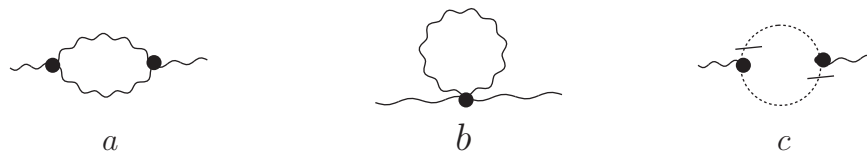


FIG. 1: Superficially linearly divergent diagrams contributing to the two-point function of the gauge field.

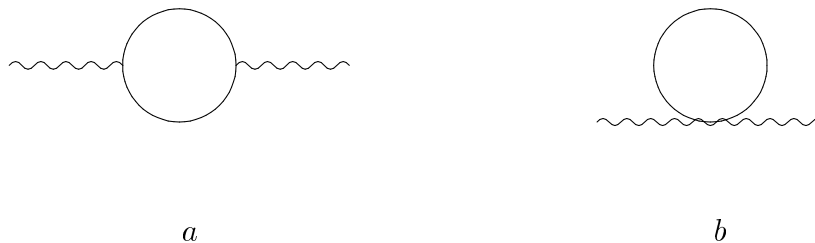


FIG. 2: Superficially linearly divergent diagrams contributing to the two-point function of the gauge field: matter sector



FIG. 3: Diagrams contributing to the two-point function of the matter field.